

Deterministic loading of an individual atom: Towards scalable implementation of multi-qubit*

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We report the realization of a deterministic single-atom preparation by the method of all-optical feedback. Using a fast-real-time feedback, the light-induced atom desorption effect and blue detuned light-induced atom collision process can increase a success probability of single-atom preparation up to more than 99%. We investigate the dynamics of loading single atom trapped in a trap with a size of hundreds of micrometers into a pair of microscopic tweezers. The detailed experimental results show that the feedback loading is spatially insensitive, which implies that it is possible to use the feedback protocol to simultaneously implement the loading of large number of qubits arrays.

Keywords: single atom, microscopic optical dipole trap, atom heating

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1. Introduction

Quantum computer and quantum information storage are built up from multiple microscopic physical systems.^[1–8] Single neutral atom trapped by optical trap offers robust quantum coherence and controllability, which provides an attractive system for simulating the complex problems in microscopic physics.^[9–11] Single atom trapped in a microscopic optical trap can serve as single quantum bit.^[12–14] Many qubits may serve as quantum registers or quantum gates.^[15,16] Scalability of optical trap to many qubits relies on multiplexing single-atom traps using diffractive optical elements or lens arrays.^[17–19] Actually implementing essential quantum device requires many qubits stored in individually addressable atoms to be quickly initialized, which means deterministic single-atom loading in all sites of an array. Although possible solutions as well as experimental capabilities are being actively developed,^[18,20–23] until now, however, controlled loading of single atom into an optical microtrap is still a challenge.

A number of different approaches to this problem have been explored including the development of collisional blockade technologies and many-atom entangled states created by Rydberg blockade. Stochastic loading of single atom in a magnetic optical trap (MOT) from a background vapor is governed by Poisson statistics,^[24–26] which can be improved by feedback control.^[27,28] For preparing single-atom in an optical microtrap, a possible solution is to use collisional mechanism. When the trap volume is small enough, a “collisional blockade mechanism” that locks the atom number at either zero

or one,^[11] where the loading efficiency was limited to 50% due to induced collisions by red detuned light. This situation can be improved using controlled collisions by blue detuned light,^[29] where the optimized loading efficiency shows a maximum success probability of $P_1 \sim 82.7\%$. Of course it may be sufficient to simply register. The probability of loading each of N sites with one atom thus is scaled by P_1^N . Approaches based on controlled induced collisions may be difficult to simultaneously load single atom in many sites of an array. Actually building such a collision process requires that the hyperfine state of trapped atoms should be accurate and fast controlled, while the initial number of trapped atoms should be small to satisfy the requirements for the lifetime of hyperfine state and collision efficiency. An alternative approach is to use feedback control. The feedback protocols are central to most classical and quantum control procedures. A controller compares the signals measured by a classical and quantum sensor with the target value. It then adjusts an actuator to stabilize the signal around the target value. Generalizing this scheme to stabilize a micro-system quantum system relies on real-time information from sensor measurements, which need to overcome a fundamental difficulty: the precise detection at the single particle level and an effective feedback procedure.

In this work, we demonstrate a real time feedback protocol to lock the single atom in a microscopic optical trap. This realization first enables us to achieve a deterministic source of single neutral atom for creating the single photon source. The spatial insensitive loading implies that it is possible to use the

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feedback protocol to simultaneously implement a large number of qubits in two-dimensional (2D) or in three-dimensional (3D) arrays.

2. Theoretical model

In a simple semi-classical picture, considering the collisions in a gas with a Maxwell–Boltzmann velocity distribution confined in an optical trap $\rho(v)v\sigma N/V$, where v is the velocity of collisional atom pairs, σ is the cross section, N is the trapped atom number, and V is their volume defined by the peak density volume of the Gaussian sphere. Under the condition of resonance light-induced processes, the velocity-dependent trap-loss collision rate is $\beta = \int \rho(v)v\sigma(N/V)(L_1 + L_2)d v$, where L_1 is the one-body loss rate resulting from induced collisions and L_2 is the two-body loss rate. The atomic motion is treated classically, the internal quantum states of the system are represented on a product basis of atomic and field states. Consider two atoms that approach to each other in their ground state potential, they can be excited to attractive or repulsive potential state, depending on the resonance light detuned to the transition level. In the attractive potential curve they will reach the short-range part of the potential and suffer hyperfine-changing collisions (HCC). In the presence of blue detuned laser they will reach to a repulsive excited potential, where the two atoms repel each other before they reach a distance R_G . This picture can be represented as a Landau–Zener avoided crossing of the dressed-atom picture. The two relevant states $|S+S, n\rangle$ and $|S+P, n-1\rangle$ are energetically degenerate at R_C . The coupling of these levels causes a Rabi splitting of $\hbar\Omega$ at R_C , where the system can adiabatically follow the upper repulsive potential curve, or undergo a transition to the lower-lying curve. The transition probability is given by the Landau–Zener formula:^[30] $P_{LZ} = \exp(-\pi\hbar\Omega^2/2D'v_C)$, where $D' \approx 3C/R_C^4$ is the difference between the slopes of the potential curves and v_C is the relative velocity of the colliding partners at R_C . If transition occurs, the atoms approach to each other more closely and HCC process may occur, which includes the two-atom loss with probability η . For an atom pair reaching R_C , the loss probability is $P_{Loss} = \eta P_{LZ} + P_{LZ}(1-\eta)(1-P_{LZ}) + (1-P_{LZ})P_{LZ} = \eta P_{LZ} + (2-\eta)P_{LZ}(1-P_{LZ}) = P_{HCC} + P_{Ind}$.

At high blue detuned laser intensities, the probability of HCC P_{HCC} is totally suppressed. The outgoing flux emerges at the $|S+S, n\rangle$ state, both atoms recede along the same ground state potential as the entrance channel potential. This procedure repeats N times until the quasi-molecule dissociates in one ground-state and one excited-state atom with kinetic energy equal to the blue detuning. By employing near resonance blue detuned light the released energy can be controlled in each inelastic collision which induces single atom

loss P_{Ind} . The maximum probability obtaining one atom is given by $P(1|2) = (1 - P_{LZ})^{N-1}P_{LZ}$ for N repetitions. The minimum time of induced single atom loss is given by $\tau = (2N - 1)gmv/D'$. If $1/\beta \gg \tau$, the collision time is approximately equal to $1/\beta$. Consider an experiment in which inelastic light-assisted collisions between a pair of trapped atoms can have two possible outcomes: both atoms are lost with probability $P(0|2) = q$; only one of the atoms is lost with probability $P(1|2) = P_1 = 1 - q$. For a larger initial number, we can assume that the light-assisted collisions persevere until either zero or one atom. In $(n + 1)$ atoms, a series of atom–atom collisions lead to the consequence that either one atom is in the optical trap or all the atoms are lost with probability $P(1|n+1) = P(1|n) + P(0|n)$. This recursive relation leads to: $P(1|n) = 1 - q + q^2 + \dots + K(-1)^n q^n$. For $n \rightarrow \infty$ the series converges to $P(1|n) = 1/(2 - P_1)$. Note that the convergence is slower for larger P_1 . Optimized loading efficiency shows a maximal success probability of $P(1|n) = 1/[2 - (1 - P_{LZ})^{N-1}P_{LZ}]$. In principle, it is difficult to achieve perfect single-atom loading by using only induced collision.

A combination of feedback ideas and single-atom-counting may be efficient over the problem. Even with the above imperfections of loading and collision under control there is still such a method more significant to initialize the loading on demand. Building such a protocol requires only fast loading process and controlled collision process. We design an automatic compensation identification system to control the loading states by using the light-induced atom desorption effect (LIAD), and to control the collision process by using blue detuned light-induced atom collision (LIAC).

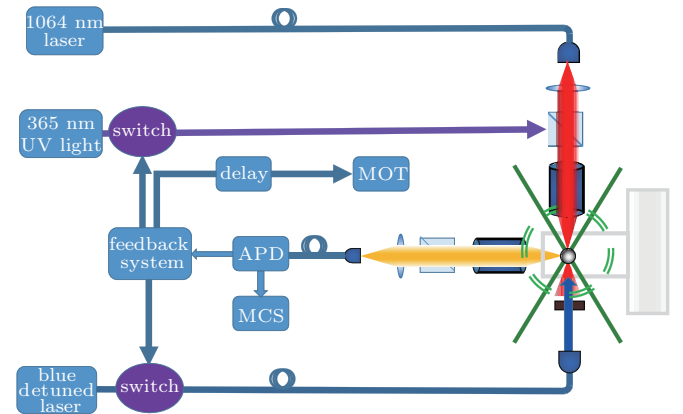


Fig. 1. (color online) Schematic diagram of the feedback set-up. The optimal geometrical overlap of optical trap with a radius of $2.3 \mu\text{m}$ with an MOT trapping volume can be achieved by light shift of trapped atoms. Light induced fluorescence (LIF) photons of trapped atoms are observed with an avalanche photodiode (APD) in single photon counting mode with high signal-to-noise ratio (SNR). Part of the signal is then fed to a control system which discriminates the different characteristic states of trap. Ultra-violet light is employed to control the loading rate of trap in range of $1-10^5$. Blue detuned laser is used to achieve the controlled collision of trapped atoms in which only one of the collision partners is lost. The MCS is the multi-channel scaler for analyzing the fluorescence of the atoms in the trap.

3. Experimental result and discussion

The schematic diagram of the experimental setup is shown in Fig. 1. In our MOT system, the two pairs of beams in horizontal plane intersect with the vacuum glass cell at an angle of 60° . For magnetic field of the MOT, the supplied current of 20 A yields a gradient of magnetic field of ~ 288 Gauss/cm. The microscopic optical trap is formed by focusing a laser beam of single-frequency 1064 nm laser. The trap depth is 1mK with a laser power of ~ 31 mW. The fluorescence photons of the trapped atoms are collected by a lens assembly with a numerical aperture of 0.29.

The desired number of atoms in an optical trap can be directly locked by the feedback system acting on the loading and collision process. The feedback system starts by fast loading cold atoms from MOT into optical trap by LIAD. Actually in order to minimize the loading time it requires the system to work at a large loading rate, which leads to the sudden many-atoms loading. Under the condition of more than one atom in trap, the control system will discriminate the state and prepare the atoms in a typical quantum state and induce an LIAC. The sequence repeats until remaining atoms left in the trap are locked to one. The loading process is controlled by LIAD which allows fast switching and on demand all-optical generation of background atoms. The optical

trap loading is well described by $dN/dt = R \exp(-\gamma_{\text{MOT}} t) - N \Gamma_L - \beta \int n^2(r, t) dr$, where γ_{MOT} is the loss rate of the MOT atoms, Γ_L and β characterize the density-independent and density-dependent losses. Loading rate of optical trap $R \approx 0.5V^{2/3}v^4(m/2kT)^{3/2}A/D$, depending only on desorption rate D and absorption coefficient rate A , can be varied by ~ 5 orders of magnitude. By controlling D the loading rate is increased according to a rate equation $D(1 - e^{-At})/A$. After shutting down the desorption light the pressure can restore to equilibrium quickly. When single atom is loaded into microtrap, the loading rate of the trap decreases to zero in about 3ms with the help of feedback system.

The controlled collision process can be understood through the simplified two-level model illustrated in the dressed state picture. The description of the dynamics of this process is shown in Fig. 2(c). Using the blue detuned light for inducing collisions, the energy released in each inelastic collision is controlled by detuning δ of the single atom resonance, so that only one of the collision partners is lost in each inelastic collision until single atom is in trap. The collision time depends on the Rabi splitting of $\hbar\Omega$ of these levels coupled by the blue detuning laser. Using the optimized parameters in our experiment, the LIAC gives a single-atom success probability of 80%.

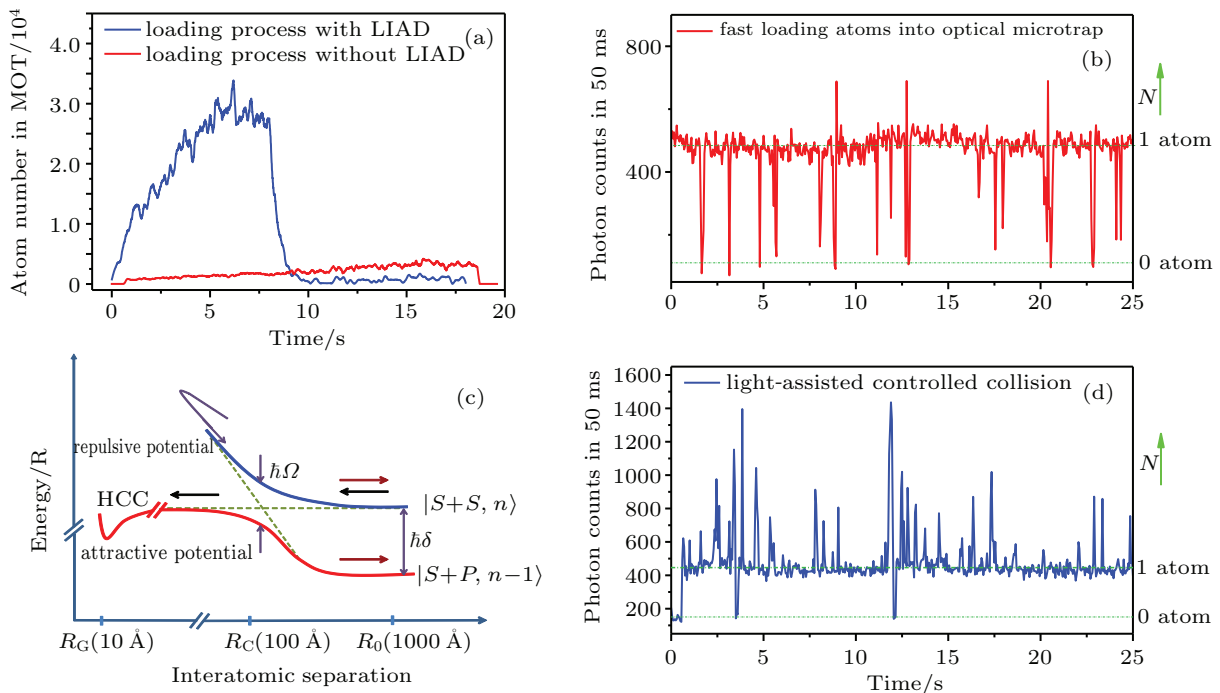


Fig. 2. (color online) Controlled single atom loading. (a) Using LIAD to control the loading states of MOT. (b) Fast loading cold atoms into an optical microtrap by using feedback approach. (c) Light induced atom collisions in the dressed-atom picture. (d) Using LIAC to achieve single atom in optical microtrap. By manipulating LIAC and LIAD, loading and loss process can be conveniently controlled.

Running feedback protocol depends on the rapid control of the loading and collisional process. An increase of the ultra-violet light power can increase the desorption rate D and minimize the loading time. The dependence of loading time on

ultra-violet light intensity is shown in Fig. 3(a). The LIAC time depends on the intensity of induced laser, velocity distribution of trapped atoms and trap volume, which is much smaller than the trap lifetime. To obtain a reasonable loading

efficiency, the value of the ultra-violet light and the parameters of blue detuning laser need to be optimized. In our case, the ultra-violet light intensity is $\sim 5 \text{ mW/cm}^2$ and the intensity of blue detuning laser is $\sim 11.5 \text{ mW/cm}^2$, where typical loading time is about 163 ms and collision time is about 194 ms. Actually, in experimental process the qubits are allowed to be initialized in several hundreds of milliseconds. In addition, the position probability distribution for cold atoms in MOT is approximately Gaussian, even for a few or one atom. This case is observed as shown in Fig. 3(b), for the case of relatively large displacement from the MOT trap center, a long loading time is needed. The case can be improved using feedback control, where the loading process of optical trap shows a nearly constant time in a large displacement region of space. The space-independent loading process means that it is possible to use the feedback system for ensemble microtrap traps within a few ten micron-sized region of space.

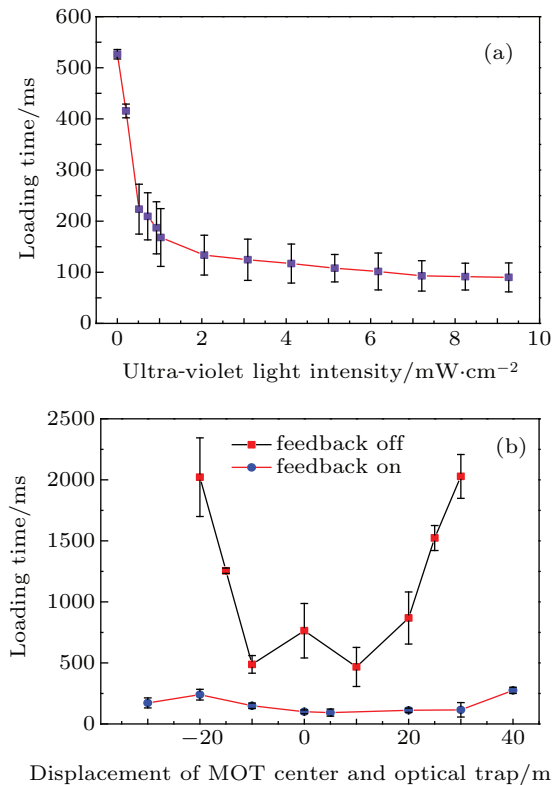


Fig. 3. (color online) Loading time as a function of the ultra-violet light intensity and spatial distribution of traps. (a) The black squares show dependence of the loading time of obtained one atom on ultra-violet light intensity. The loading time decreases to ~ 100 ms which is in practice limited by sizes of two traps with a contrast of ~ 3 orders of magnitude. (b) The red points show the dependence of the loading time of obtained one atom on displacements between the trap centers, where the time is dominated by spatial probability distribution of single atom in MOT. The blue solid circles show a decrease of the loading time to a constant offset for a large range of spatial distribution of traps intensities by employing feedback control. Error bars represent the statistic results.

Figure 4 shows typical probability measurements of single atom in an optical trap. Without feedback, atoms are stochastically loaded and lost in the microscopic optical trap,

resulting in a random walk in trap occupation number. The probability of single atom in trap is limited to a maximal value of $\sim 54.4\%$. Our optical trap volume is not so small that the “collisional blockade” mechanism does not become the dominant loss mechanism as soon as there is more than one atom in the trap.^[11] In order to improve the probability of single atom in trap, we run the feedback program. When the initial atom number trapped in optical trap is less than one, the loading rate of MOT is rapidly increased by LIAD until one atom is trapped. When the number of trapped atoms is more than one, the MOT is turned off and the repumping laser becomes strong to ensure that the trapped atoms are in the typical ground state, while the blue detuned light is turned on to induce a controlled collisions in trapped atoms. After several hundreds of milliseconds the MOT lasers will be turned on again to count the number of atoms trapped in optical trap. The feedback-on data in Figs. 4(b) and 4(d) are representative of the behavior under near-optimal conditions for our experimental configuration, and correspond to a probability $P_{1\text{atom}}$, $\sim 99.9\%$, of finding a single atom in the trap. With feedback control, the fluorescence is nearly constant at the single atom level, with occasional fast dips to a background level and spikes to the N -atom level. The fluorescence level statistics depends on the time bin size. We show an optimal size of the time bin (100–200 ms) for minimizing the probability of indeterminate atom number while providing accurate information for the feedback system.

Scalability of optical trapping to many qubits relies on either multiplexing traps using diffractive optical elements or lens arrays. The difficulty in achieving multiplexing traps by using diffractive optical elements or lens arrays is that the trapping region of space is large enough to provide a few hundreds of traps and the trap volume is still small enough to ensure that strong collisions are blocked. A more serious difficulty is that the inhomogeneous intensity profile of a focused Gaussian laser beam is position-dependent, which leads to a little different trap depths for the distribution of microlenses in space. Since none of the trap depths are identical, it is difficult to achieve high single atom trap noly using LIAC process. In our scheme, the solution of deterministic single-atom loading on N sites of an array requires the atom signal to be locked to the value N , which depends on the trapping lifetime, the space-independent feedback parameters and the ability to count the single-atom number.

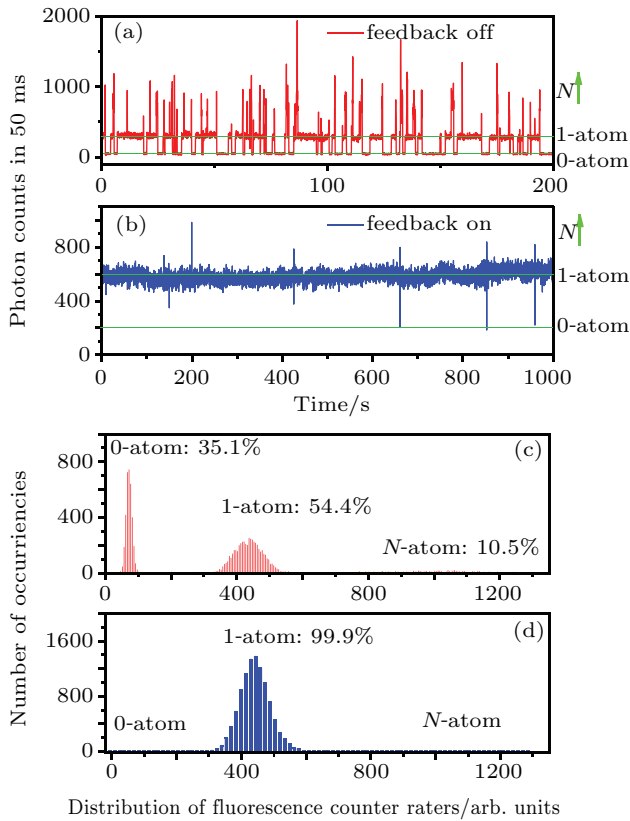


Fig. 4. (color online) Preparation of single atom in optical microtrap. (a) Stochastic loading of atoms in a microtrap from MOT. (b) Histogram displays the 0-atom, 1-atom, and N -atom peaks. The distribution of atom number in the steady state is governed by light-assisted collisions and random one-body events. (c) and (d) Feedback protocol for a single-atom preparing. The 0-atom and N -atom peaks are clearly missing. Both histograms correspond to a total of about 10000 counting samples.

First, multiple traps may need much longer loading and collision time, and high vacuum is needed to ensure a long vacuum-limited lifetime. Meanwhile, the starting point for a statistical loading process should be considerably enhanced to implement the rapidly initial loading process. The LIAD process relies on the fact that atoms adsorbed at the walls of the vacuum chamber can be desorbed by the irradiation of short wavelengths, which allows fast switching and all-optical generation of controlled atomic density. After shutting down the desorption light, the pressure can be restored to equilibrium quickly, so that it is possible to achieve a long trapping lifetime after implementing the LIAD process. Second, for a large position distribution of traps, realizing the loading of trap arrays will require the feedback system to be space-independent. Here, we present the simulation of a set of optical trap arrays by implementing the feedback system to stabilize the spatially separated traps, and the simulation results are shown in Fig. 5. That the high loading probability is independent of position distribution of the traps mean that the deterministic single atom loading region of $30 \mu\text{m} \times 30 \mu\text{m} \times 30 \mu\text{m}$ can be achieved. We also present the implementation range from several small traps to large-scale traps in three-dimensional (3D) space as shown in Fig. 5(b). One of 3D architectures is over ~ 25 times larger than the size of the optical micro-

trap. It is evident that based on our feedback scheme, the loading region of space with high single-atom probability is large enough. In our experiments the maximal length of space is limited by MOT size. The trap size of the MOT is in a range of $110 \mu\text{m} \times 103 \mu\text{m} \times 200 \mu\text{m}$, depending on the magnetic field gradient. Third, accurate single-atom resolution in microscopic trap arrays is necessary for performing feedback. Our signal-to-noise ratio of detected single atom could be improved by developing a digital low pass filter. By investigating the primary noise sources, Hume *et al.* obtained single-atom resolution for atom numbers as high as 1200.^[31] Their approach may be one of promising approaches.

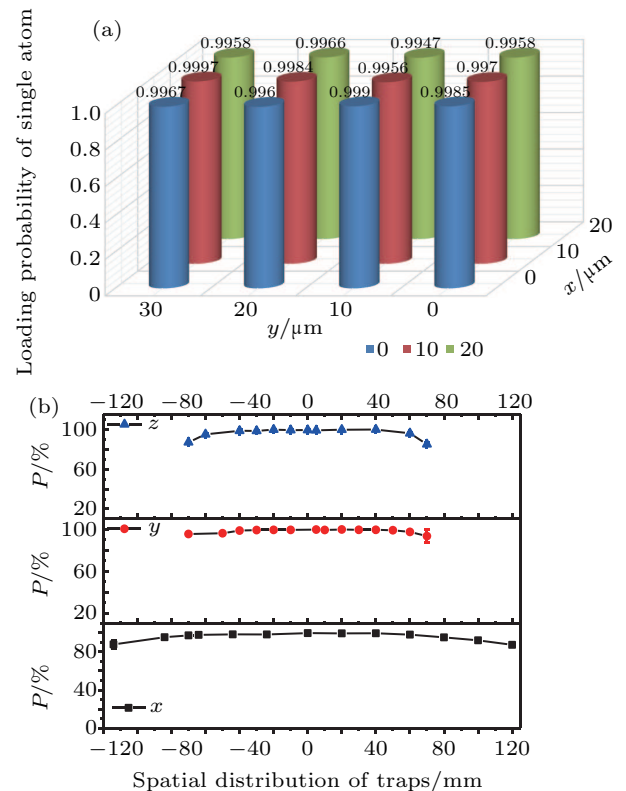


Fig. 5. (color online) Loading probability of single atom in optical microtrap as a function of spatial distribution of traps. (a) Experimental simulation of preparing single atom in a two-dimensional optical microtrap. Individual traps in these patterns have a close to unity in efficiency with lateral separation of $10 \mu\text{m}$. Error bars represent the statistic results. Each point of experimental data is from the accumulation of at least 150 sequences. (b) The implementation range from several small traps to large-scale traps in three-dimensional space.

4. Summary

In this work, we demonstrate a real time feedback protocol to lock the single atom in a microscopic optical trap. This realization first enables us to achieve a deterministic source of single neutral atom for creating the single photon source. The spatial insensitive loading implies that it is possible to use the feedback protocol to simultaneously implement a large number of qubits in 2D or in 3D arrays.

Loading the trap arrays in a wide range of space dimensionalities needs further studying experimentally. Note that

actually for a set of microscopic traps immersed in a large-sized MOT, due to the extremely strong LIAD, the initial load of traps is fast, which means that the systems does not need to run feedback on the traps one by one. A combination of control of MOT magnetic field and desorption rate D , can improve the loading rate by ~ 10 orders of magnitude. A combination of a single-loss collisional redistribution process^[32] and feedback control may eventually prove that the loading efficiency is substantially improved. The reality of the situation can be established by the act of feedback protocol, which forces the array system instantaneously to enter into occupancy of a qubit state. Therefore, this deterministic single-atom preparation in single microtrap needs to be accounted for when developing a feedback protocol to stabilize ensemble traps. This setting manipulates individual quantum systems by classical feedback, which not only arouses a fundamental interest in the external motional degrees based on single photon measurement, but also offers a promising route to generating interesting deterministic quantum states based on non-classical measurement mechanisms. Barredo *et al.* used atom-by-atom assembly to implement a deterministic preparation of regular arrays of individually controlled single atoms.^[33] These results open up exciting prospects for quantum engineering with neutral atoms in tunable 2D geometries. In our approach, we perform a fast real-time feedback the LIAD and LIAC process giving a success probability of single-atom, more than 99%. Our idea is simple, yet a fairly effective solution to the implementation of the single-photon source.

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